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EVALUATION OF MONOTUBE SIGN SUPPORT STRUCTURE

State of the Art

Final Report

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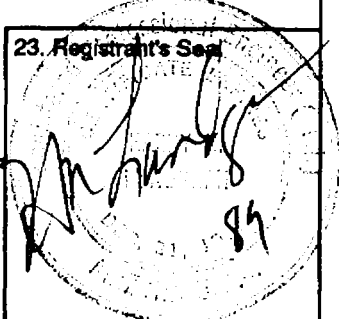
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16. Abstract The existing design criteria and other relevant publications for bridge type sign structures are evaluated and critiqued. Static and dynamic stresses and deformations are investigated for two common types of monotube construction. Both along-wind and cross-wind behavior are investigated. Several levels of improved design procedure are formulated in the report that more appropriately articulate the design requirements than the existing specifications. The implementation of the recommendations contained in this report make possible the utilization of more economical sign structures than are considered acceptable under the existing specification.					
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1. INTRODUCTION

A. PROBLEM STATEMENT

Truss-type sign support structures have been used and performed satisfactorily; however, they are not economical. Recently, engineers have begun to use monotube sign support structures which are more economical. The current design criteria and specification (1) do not address the design of monotube structures (neither span-type nor cantilever structures) adequately. Furthermore, manufacturers of monotube structures have their own design procedures and use different material. Limited field tests on monotube span-type sign support structures have been conducted under previous research; however, sign support cantilever structures have not been studied. The intent of this study is to evaluate the design and the use of monotube sign support structures for both span-type and cantilever structures. The findings of this study will provide the foundation for the development of improved design criteria.

B. RESEARCH OBJECTIVES

The objective of this study is to evaluate the current design and practice of monotube sign support structures, both span-type and cantilever structures. Based on the presently available information and the state of the art, the study will address the response of the structure to the wind loads, the

influence of material types and cross sectional shapes of the monotubes, and the long-term service characteristics of the structure.

To accomplish this objective, the following tasks will be conducted:

1. Review relevant current domestic and foreign practice and research findings. This information shall be assembled from the technical literature and a canvass of other states' designs and practices of monotube sign support structures.
2. Analyze and evaluate the information obtained in Task 1. The evaluation should analyze concepts, assumptions and limitations of each type of design method and practice including static and dynamic stresses, deflections, dynamic characteristics, and structural resonances.
3. Identify the shortcomings, if any, in current design and practice of the monotube sign support structures. Prepare design methods(s) and procedure(s) based on the present state of the art.
4. Prepare a final report to include recommendation and detailed information that can be used to implement the research results. If additional research is needed, a detailed work plan and estimated budget and manpower will be developed.

II. REVIEW OF PROBLEM

A. EXISTING AASHTO CRITERIA^{1,2}

The design of sign support structures is governed by AASHTO "Standard Specifications for Structural Supports for Highway Signs, Luminaires and Traffic Signals² (1985) developed by the AASHTO Subcommittee on Bridges and Structures. Each relevant section will be identified and discussed.

1.2.5 - Application of Wind Load. This section provides the wind pressure computation equations and tabular values for adapting the reference values to the specific site conditions. It also specifies the Wind Drag Coefficients for various cross-sections including the circular cross-section of interest in this study. Wind load isotach plots for various return periods are published herein.

COMMENTS: This criterion agrees in sense and value with similar requirements in ANSI A58.1-1982³ and other references. The material is based on sound scientific principles and has wide acceptance.

The discussions concerning Reynolds numbers are needlessly complex. It is simpler to merely take the worst case for drag coefficient than to attempt to arbitrarily select an appropriate Reynolds number and subsequently an arbitrary drag coefficient.

Updated isotach values appear in ANSI A58.1-1982³.

1.9.1 - Deflection. This section stipulates that "Overhead sign structures (span type) shall be proportioned to avoid resonance at critical wind speeds by limiting their vertical deflection". This has generally been accomplished by using the value $d^2/400$ in feet as a limit for dead load deflection where d is the sign depth in feet. This requirement is being deleted in a code revision.

COMMENTS: It is because of this stipulation, which was felt to be unreasonable and perhaps irrelevant, that the previous study^{6,7} and this subsequent review have been undertaken. For example, this requirement can be circumvented by merely making the sign depth large enough to provide a sufficiently large deflection. This provision will be discussed further in the review of the Pelkey paper⁴.

1.9.6 - Vibration and Fatigue. This section provides a means by which the cross-wind resonating frequencies due to vortex shedding of cantilever structures can be calculated.

COMMENTS: The procedure described herein neglects the lock-in phenomenon and predicts a single value for critical velocity. In addition, a procedure for computing stresses resulting from cross-wind oscillation is presented for cantilever type sign structures. This procedure is based upon the Standard Wind Pressure adjusted by certain factors. This section may have some value in calculating fatigue effects.

The following sections are from the accompanying Commentary on Standard Specifications for Structural Supports for Highway Signs, Luminaires, and Traffic Signals.²

1.2.4 - Wind Load. Discusses the basis of the wind speed maps.

1.2.5 - Application of Wind Load. Provides a general discussion of the background of the fundamental wind pressure equation including the basis of the gusting, elevation, and drag coefficients. A discussion of the relationship of Reynolds numbers to the drag coefficient is included. A fairly extensive explanation of the effect of wind on span wire structures is included but no discussion of rigid type of span structures.

COMMENT: Again, the extended concern with determining the Reynolds numbers seems unwarranted.

B. PERFORMANCE OF EXISTING STRUCTURES

There does not seem to be a history of problems with the span (or bridge) type monotube structures. No reports of poor behavior could be located. The cantilever sign structure has exhibited problems, although not apparently wide-spread. These are reported to be fatigue-based.

C. COMPARISON OF CONFIGURATIONS

There are two configurations of span-type monotube structures in common use. The first is comprised of tapered elements. The vertical support members are tapered uniformly from base to top. The horizontal beam member is tapered uniformly from a maximum at mid-span to a lesser diameter at the ends where it is supported by the vertical members. The connection at this point is designed to resist bending in the vertical plane but only resists minimal bending out of plane. This is a somewhat standardized "product-type" structure manufactured by several firms and utilized in many parts of the country. The origin of the rationale for this design is not clear but it is suspected that the connection was intended for easy field erection and the tapered members for more economical utilization of material.

The second configuration of span-type monotube sign support structure has a constant outside tube diameter and has a continuous radial transition from vertical support to spanning horizontal member. The wall thickness of the three elements (vertical member, radius transition member, and horizontal spanning member) can be varied to utilize material more economically. This system provides bending resistance in all planes at all points.

III. REVIEW OF PREVIOUS RELEVANT PUBLICATIONS

Following are discussions of publications that provide background relevant to the subject of this investigation. Additional references can be found in these publications and will not be reprinted here.

A. LONG SPAN OVERHEAD SIGN STRUCTURES BY R.E.PELKEY⁴

This paper, published in 1971, correctly criticizes the AASHTO stipulation of $d^2/400$ as a means of providing adequacy against cross-wind oscillation. The paper does not, however, suggest an alternative procedure.

Because of it's significance as the genesis of this investigation, a similar development with associated commentary follows.

The AASHTO specification is based on the idea that vortex-shedding resonance from a flat panel highway sign should not occur before the wind speed against the panel reaches 80 miles per hour. This can be viewed in the following way:

1. Vortex Shedding

The Strouhal relation is $\frac{nd}{v} = S$

where d = depth of the sign (cross-wind)
 v = 80 mph = 117.33 ft/sec
 S = Strouhal Number

Thus the frequency is

$$n = \frac{vS}{d} = \frac{117.33 S}{d}$$

2. Beam Natural Frequencies

a. For a pin-ended uniform beam

$$n = \frac{\pi}{2} \sqrt{\frac{EIg}{wL^4}}$$

where EI = beam stiffness

g = gravitational constant (32 ft/sec)

w = weight per unit length

L = length

b. For a cantilever uniform beam

$$n = 0.56 \sqrt{\frac{EIg}{wL^4}}$$

3. Beam Deflections (under beam self-weight)

a. For a pin-ended uniform beam

center deflection $\Delta_p = \frac{5}{384} \frac{wL^4}{EI}$

b. For a cantilever beam

tip deflection $\Delta_c = \frac{wL^4}{8EI}$

The analysis proceeds as follows: The frequency of the beam and of vortex shedding are assumed to coincide at 80 mph, i.e.,

$$\begin{aligned}\frac{117.33 \text{ S}}{d} &= \frac{\pi}{2} \sqrt{\frac{E I g}{w L^4}} && \text{(pinned case)} \\ &= 0.56 \sqrt{\frac{E I g}{w L^4}} && \text{(cantilever case)}\end{aligned}$$

The expression $\frac{E I}{w L^4}$

is evaluated from the beam deflections:

$$\begin{aligned}\frac{E I}{w L^4} &= \frac{5}{384 \Delta p} && \text{(pinned case)} \\ &= \frac{1}{8 \Delta c} && \text{(cantilever case)}\end{aligned}$$

In the pinned case:

$$\frac{E I}{w L^4} = \left[\frac{2 \times 117.33 \text{ S}}{\pi d \sqrt{g}} \right]^2 = \frac{173.27 \text{ S}^2}{d^2}$$

In the cantilever case:

$$\frac{E I}{w L^4} = \left[\frac{117.33 \text{ S}}{d (0.56) \sqrt{g}} \right]^2 = \frac{1363.28 \text{ S}^2}{d^2}$$

Hence, in the pinned case:

$$\frac{173.27 \text{ S}^2}{d^2} = \frac{5}{384 \Delta p}$$

$$\text{or } \frac{1}{\Delta p} = 13307 \left(\frac{S^2}{d^2} \right); \Delta p = 7.5 \times 10^{-5} \left(\frac{d^2}{S^2} \right)$$

and in the cantilever case:

$$\frac{1363.28 S^2}{d^2} = \frac{1}{8 \Delta c}$$

$$\text{or } \frac{1}{\Delta c} = 10906 \left(\frac{S^2}{d^2} \right); \Delta c = 9.2 \times 10^{-5} \left(\frac{d^2}{S^2} \right)$$

If the composite of the Strouhal numbers for a flat plate (.145) and for a circular cylinder (.20) is taken as .18, then the resulting calculations

$$\Delta p = 2.315 \times 10^{-3} d^2$$

$$\Delta c = 2.840 \times 10^{-3} d^2$$

approximates the AASHTO requirement of $d^2/400$,

$$\Delta = 2.5 \times 10^{-3} d^2$$

If, instead, the Strouhal number of a plate (.145) had been used, a requirement of $d^2/250$ would have resulted. In summary, the criterion is neither rigorous nor realistic. The turbulence caused by flat plates (signs) will tend to dampen and thus diminish the effects of vortex shedding.

B. 1983 ONTARIO HIGHWAY BRIDGE DESIGN CODE AND COMMENTARY⁵

This document is a voluminous and very detailed design specification covering all aspects of highway structure design. Especially sections 2-4.5.4.4 on page 44 of the Code, Section A2-2 page 72 of the Code, and CA2.2 page 65 of the Commentary detail the computation of vortex frequencies and the associated resonance problems. A requirement is made that an analysis be performed for all modes of vibration for which the natural frequency falls below a designated value (a function of the Strouhal number, the diameter of the tube, the 50 year return wind pressure, and the exposure coefficient). In addition to the information on cross-wind behavior, the Code covers the computation of normal wind pressures and the resulting stresses.

C. "STATIC AND DYNAMIC BEHAVIOR OF MONOTUBE SPAN-TYPE SIGN STRUCTURES" BY EHSANI, CHAKRABARTI, AND BJORHOVDE⁶

This investigation (funded by the Arizona Department of Transportation) primarily addresses the problem of the span-type structure. The study was limited to the tapered configuration of tubular structure described earlier in this report. Various span/height configurations were analyzed for both static and dynamic behavior. Their computations show that you must use a three-dimensional analysis to pick up all of the natural frequencies since some (including the first) have out-of-plane contributions. An apparent units error

caused the frequency calculations to be in error by as much as a factor of two. However, these values do not appear to have been used in any of the calculations upon which the conclusions have been drawn.

A series of Conclusions and Recommendations are listed in volume 1. Some of the most significant as they relate to this study are:

1. "The stress levels associated with the actual deflections are well below the magnitudes of the allowable stresses, even though the maximum deflection is much larger than the $d^2/400$ level."
2. "The design of monotube structures is governed by stiffness (i.e., deflection) rather than strength (i.e., stress) criteria. Allowable stresses were never exceeded, even though large deflections were recorded."
3. The dynamic deflection increments that were produced by the vortex shedding were one order of magnitude (or more) less than the static values for almost all wind speeds. Their influence on stress and deflection levels is therefore not important, since overloads are generally acceptable for short-time loads on ductile structures. (It is noted that vortex shedding produces in-plane displacements when the wind acts perpendicularly to the plane of the structure.)"

4. "It is believed that the resonance condition is not serious, for several reasons. They are as follows:
 - (a) Wind speed must be maintained within a narrow range and for a prolonged period in order for resonance to occur.
 - (b) The structural analysis was performed on the assumption that the monotube structure does not possess any damping capability.
 - (c) The apparent strength of the material increases with the rate of loading."
5. "Analysis and design can follow the criteria of any suitable design specification. Only static evaluations are needed, and the in-plane and out-of-plane responses can be dealt with independently.

Several statements in section D. above can be questioned. The "lock-in" phenomenon⁸ whereby exciting frequencies in a fairly broad neighborhood of the computed natural frequencies of the structure can cause the structure to resonate is contrary to the statement in section 4(a) to the effect that wind "must be maintained within a narrow range and over a prolonged period in order for resonance to occur". Also, the neglect of

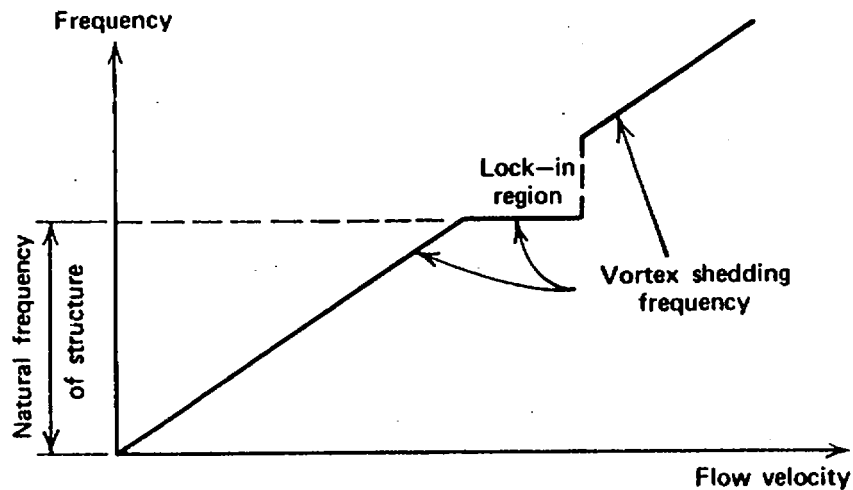


Figure 1

damping in the computations appears to be unjustified. It can be questioned that "apparent strength of material increases with rate of loading" when thought of in conjunction with a fatigue-sensitive problem.

Section 7.4, Proposed Analysis Procedure on page 82 of Volume 1 outlines a design sequence. The sequence follows usual and accepted procedures with the addition that "Gravity load deflection magnitudes should be compared to the span of the monotube structure that is being designed/evaluated: deflection-to-span ratios of 1/150 or less are acceptable

from strength as well as serviceability standpoints." It is understood that this is purely an aesthetic requirement and is not similar to the $d^2/400$ limitation set in AASHTO as an attempt to limit the behavior due to vortex shedding.

Under section 8.2 Recommended Future Studies several items are listed. The most important relevant to this study was:

Analytical Modeling and Field Testing: .."Extensive field monitoring of monotube structures is needed ... the analytical study has been limited to wind speeds of approximately 27-29 mph"

This is severely limiting and those Conclusions and Recommendations (partially listed above) that may be based upon related results must be questioned as, in fact, it is implied in the justification for field testing under this same section (8.2).

D. "FIELD TESTING OF MONOTUBE SPAN-TYPE SIGN STRUCTURES"⁷, BY MARTIN, EHSANI, AND BJORHOVDE

This study involved a correlative investigation of two span-type structures. Two existing structures of the tapered type were instrumented and also modeled in the computer. The results were then compared. Wind speeds and corresponding deflections were measured on the structures. It should be noted that maximum wind velocities at both sites were about 23 mph.

The relevant (to this study) Summary and Conclusions from the report can be digested as follows:

1. The field and computer data showed good correlation.
2. The full scale structures did not meet the $d^2/400$ criterion.
3. Stress levels were well below the (non-fatigue) allowables.
4. Resonance did not occur in the field testing.

The basis of the conclusion in item 4. of the preceding paragraph may be weakened by the previously noted error in determining natural frequencies with the computer model.

E. "WIND EFFECTS ON STRUCTURES"⁸, SECOND EDITION, BY E. SIMIU AND R.H. SCANLAN

This book contains the background theory and much of the application concepts important to the subject of this study. Of especial interest are the sections on Bluff-body Aerodynamics, Structural Dynamics, Aeroelastic Phenomena, and Wind Tunnels. These fundamentals are the necessary background for this study and since it is readily available the development of this material will not be reproduced here.

IV. WIND EFFECTS ON SIGN STRUCTURES

A. ALONG WIND BEHAVIOR

1. Drag Forces. Drag force calculations are quite well defined for both the flat plate (sign) and the cylinder (the structure). The drag on circular sections is dependent upon the applicable Reynolds number which in turn is a function of the wind velocity (U), the typical surface dimension (L)(in the case of a cylinder it is the diameter), and the kinematic viscosity (ν).

$$Re = U L / \nu$$

The following figure⁸ shows the relationship of

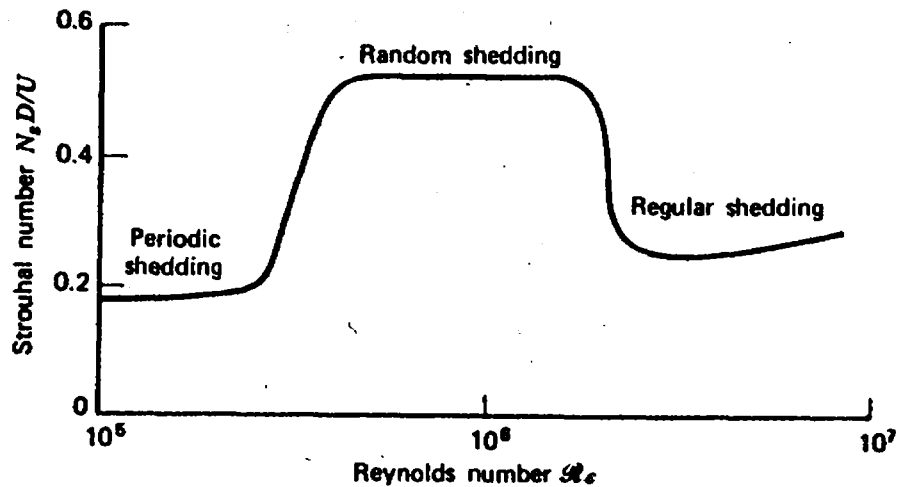


Figure 2

the drag coefficient, C_D , to Re and, since Re is linearly dependent upon U , the plot can also be thought of as a plot of wind speed versus the drag coefficient. It can be seen that the drag coefficient is largest at the lowest velocity and drops off quickly before it again begins to rise. The point is that all of these values and the drag coefficient for a flat plate are well known and widely published^{5,8}, therefore the determination of the wind forces on the structure for static force analysis are well defined. These forces can be applied and an evaluation of the adequacy of the structure performed. As mentioned previously, it is recommended that it not be attempted to determine a rational Reynolds number, but rather the maximum drag coefficient, C_D be used for design purposes.

2. Effect of Sign Locations. For static wind load analysis the forces on the structure will be greatest when the entire horizontal member is covered with the largest signs. Because of the excellent torsional properties of the tube, unsymmetric placing of signs will have negligible influence on maximum stresses. The effect of varying the sign placement on the natural frequency is discussed in the section on natural frequencies.

3. Dynamic Effects. Along wind dynamic behavior is difficult to quantify. It generally is not a problem unless something upstream is causing a pulsating wind pressure. This can be caused by vortex shedding of an upstream obstacle. This, then, becomes a function of local conditions and constraints-again difficult to solve in a generalized fashion. It is not anticipated that the structures being considered in this report would need to be analyzed for this condition. The use of the gust factor provides some protection against localized wind phenomena.

B. ACROSS WIND BEHAVIOR

Vortex Shedding Excitation of Slender Members

- a. Introduction. The types of structural members to be considered are elongated and may be either uniform or tapered. In general, vortex shedding from tapered members can occur, although the effective diameter, or cross-sectional dimension D_e , that governs vortex shedding will be some average over a portion of the member span most clearly exposed to an uninterrupted air flow. The shedding is governed by the Strouhal relation where n_s is the frequency of vortex

$$\frac{n_s D_e}{U} = S \quad (1)$$

shedding (through a complete cycle, i.e. a vortex from each side, in opposite directions of whirl), D_{θ} is the effective cross-sectional dimension alluded to above, U is cross-flow wind velocity, and S is the (constant) Strouhal number for the section shape of the structural member. (Generally, S ranges between 0.1 and 0.25; $S = 0.2$ is a good estimate for a circular section).

For a given value of D_{θ} , and constant S , it is clear from equation (1) that the frequency n_s of shedding is proportional to the cross-wind velocity U . However, this fact holds for a fixed member and does not account for the phenomenon of lock-in, which takes place when the member moves appreciably in response to the alternating pressures that accompany vortex shedding.

Such vibratory member motion becomes most pronounced when the vortex shedding rhythm coincides with a natural frequency of the member, when a sort of member "resonance" occurs. During this action the rhythm of vortex shedding actually locks in to the natural structural frequency, and member excitation at this frequency continues to occur for a considerable range of wind velocities more or less centered on the velocity that originally excites the resonant structural frequency.

The lock-in phenomenon accounts for the fact that structural members may be vibrated much more often by variable winds than might be inferred from equation (1) above.

Practically speaking, no important vibration of a structural member occurs at any frequency other than one of its natural frequencies - usually its lowest. Occasionally, if the member is slender enough, a higher mode and frequency may be excited.

b. Analysis. Although the details of vortex shedding and lock-in are complex and non-linear, it may be addressed by a linear analysis that treats the phenomenon as if it were member resonance. When vortex shedding occurs, an alternating force $F(x,t)$ is produced at station x along the slender member:

$$(F_{x,t}) = 1/2\rho U^2 C_S D_\theta(x) \sin(2\pi n_S t) \quad (2)$$

where $1/2\rho U^2$ is the dynamic cross-wind pressure, ρ is the air density, C_S is an appropriate "lift" coefficient ($C_S \cong 0.7$ for a circular member section).

Suppose that the structural member deforms in mode $\phi(x)$. Suppose further that the natural frequency n_0 of mode ϕ is exactly at the Strouhal frequency n_S ; i.e.

$$n_0 = n_S \quad (3)$$

which is equivalent to assuming that the lock-in region is under discussion.

Let $\delta(x,t)$ be the cross-wind deflection of the structural member in question, and let $m(x)$ be the member mass per unit length at that point.

The differential equation governing its vibration is

$$m(x) \frac{d^2\delta}{dt^2} + C(x) \frac{d\delta}{dt} + k(x)\delta = F(x,t) \quad (4)$$

where $c(x)$ and $k(x)$ are effective local damping and stiffness, respectively.

For mode $\phi(x)$, δ may be written

$$\delta(x,t) = \phi(x) p(t) \quad (5)$$

where $p(t)$ is a generalized coordinate in a single degree of freedom system. Using the "dot notation"

$\dot{\delta} = \frac{d\delta}{dt}$, etc equation (4) becomes

$$m(x) \phi(x) \ddot{p}(t) + C(x) \phi(x) \dot{p}(t) + k(x) \phi(x) p(t) = F(x,t) \quad (6)$$

we now multiply equation (6) by $\phi(x)$ and integrate over the member span $(0,L)$; giving

$$M[\ddot{p} + 2\xi\omega_0\dot{p} + \omega_0^2 p] = \int_0^L \phi(x) F(x,t) dx \quad (7)$$

where

$$M = \int_0^L m(x) \phi^2(x) dx \quad (8)$$

is the generalized mass of the member;

$$\omega_0 = 2\pi n_0 \quad (9)$$

$$M\omega_0^2 = \int_0^L \phi^2(x) k(x) dx \quad (10a)$$

$$2M\xi\omega_0 = \int_0^L C(x) \phi^2(x) dx \quad (10b)$$

and

ξ = damping ratio-to-critical

Writing

$$F_0 \sin(2\pi n_s t) = \int_0^L F(x, t) \phi(x) dx \quad (11)$$

where

$$F_0 = 1/2 \rho U^2 C_s \int_0^L D_\theta(x) \phi(x) dx \quad (12)$$

the solution to equation (7) is

$$p(t) = \frac{F_0 \sin[(2\pi n_s t) + \theta]}{\omega_0^2 M [(1 - \Omega^2)^2 + (2\xi\Omega)^2]^{1/2}} \quad (13)$$

where

$$\Omega = \frac{2\pi n_s}{2\pi n_0} \quad (14)$$

and θ is some phase angle, unimportant to the present discussion.

At lock-in $n_s = n_0$, $\Omega = 1$ and the maximum amplitude of $\delta(x)$ is

$$\delta(x)|_{\max} = \phi(x) P_{\max} = \frac{\phi(x) \rho U^2 C_s \int_0^L D_{\theta}(x) \phi(x) dx}{4\omega_0^2 M\xi} \quad (15)$$

but, by equations 1, 3, and 9:

$$\delta\omega_0^2 = \left| \frac{2\pi S U}{D} \right|^2 \quad (16)$$

and hence

$$\delta(x)|_{\max} = \frac{\phi(x)_{\max} \rho C_s D_{\theta}^2 \int_0^L D_{\theta}(x) \phi(x) dx}{(4\pi S)^2 M\xi} \quad (17)$$

An interesting fact about equation (17) is that it contains neither the wind velocity V , nor the natural frequency n_0 , explicitly. This is due to the assumption of lock-in.

c. Commentary. Equation (17) is the basis of recommendations made in Section A2-2.3 of the Ontario Highway Bridge Design Code⁵. There are some practical problems with equation (17) that require discussion. One is that vortex shedding cannot occur uniformly and simultaneously over a tapered span at a single wind velocity, since the span wise coherence of the local (two-dimensional) vortex shedding is not maintained. Perhaps the simplest response to this from an

engineering viewpoint is to estimate an effective value of D_θ , remove it from the spanwise integral in equation (17) and retain the effective D_θ as a factor in the form D_θ^3 .

Because equation (17) does not contain the natural frequency (n_0), in principle this value does not have to be calculated. However, the vibration mode shape that accompanies n_0 must be known, or at least estimated, since it enters the integrals $\int_0^L \phi(x) dx$ and $\int_0^L m(x) \phi^2(x) dx = M$. Hence, a reasonable approach is to make simple estimates of $\phi(x)$ and use them to calculate the above integrals. Such estimates can be made by assuming deflection shapes that satisfy the boundary conditions. Examples are:

(a) for pin-ended member:

$$(a1) \quad \phi(x) = \sin \left(\frac{\pi x}{L} \right)$$

$$I_1 = \frac{M_0}{L} \int_0^L \phi(x) dx = \frac{2L}{\pi}$$

taking $m \cong \frac{M_0}{L}$ where M_0 the total mass of the member:

$$I_2 = \frac{M_0}{L} \int_0^L \phi^2(x) dx = \frac{M_0}{2}$$

Thus,

$$\frac{I_1}{I_2} = \frac{4L}{\pi M_0} = 1.273 \frac{L}{M_0}$$

$$(a2) \quad \phi(x) = 4 \left(\frac{x}{L} - \frac{x^2}{L^2} \right)$$

$$I_1 = \int_0^L \phi(x) dx = \frac{2L}{3}$$

$$I_2 = \frac{M_0}{L} \int_0^L \phi^2(x) dx = \frac{8M_0}{15}$$

$$I_1/I_2 = 1.250 \, L / M_0$$

(b) for a cantilever member:

$$(b1) \quad \phi(x) = 1/2 [1 - \cos(\frac{\pi x}{L})]$$

$$I_1 = \int_0^L \phi(x) dx = L/2$$

$$I_2 = \frac{M_0}{L} \int_0^L \phi^2(x) dx = \frac{3M_0}{8}$$

$$I_1/I_2 = 1.333 \, L / M_0$$

$$(b2) \quad \phi(x) = (x/L)^2$$

$$I_1 = \int_0^L \phi(x) dx = L/3$$

$$I_2 = \frac{M_0}{L} \int_0^L \phi^2(x) dx = \frac{M_0}{5}$$

$$I_1/I_2 = 1.591 \, L / M_0$$

$$(b3) \quad \phi = 1/2 \left[3 \left(\frac{x}{L} \right)^2 - \left(\frac{x}{L} \right)^3 \right]$$

$$I_1 = \int_0^L \phi(x) dx = 3L/8$$

$$I_2 = \frac{M_0}{L} \int_0^L \phi^2(x) dx = \frac{33M_0}{140}$$

$$I_1/I_2 = 1.591 \, L / M_0$$

(c) For a member built in at both ends:

$$(c1) \quad \phi(x) = 1/2[4 - \cos(\frac{2\pi x}{L})]$$

$$I_1 = \int_0^L \phi(x) dx = L/2$$

$$I_2 = \frac{M_0}{L} \int_0^L \phi^2(x) dx = \frac{3M_0}{8}$$

$$I_1/I_2 = 1.333 L / M_0$$

$$(c2) \quad \phi(x) = 16 \left[\left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 + \left(\frac{x}{L} \right)^4 \right]$$

$$I_1 = \int_0^L \phi(x) dx = \frac{8L}{15}$$

$$I_2 = \frac{M_0}{L} \int_0^L \phi^2(x) dx = \frac{768M_0}{1890}$$

$$I_1/I_2 = 1.3125 L / M_0$$

Note that, on average

(a) for pin-ended members

$$\frac{I_1}{I_2} \approx \frac{1.26 L}{M_0} = \frac{1.26}{w}$$

(b) for cantilever members:

$$\frac{I_1}{I_2} \approx \frac{1.53 L}{M_0} = \frac{1.53}{w}$$

(c) for doubly built-in members:

$$\frac{I_1}{I_2} \approx \frac{1.32 L}{M_0} = \frac{1.32}{w}$$

where w is the weight per unit length.

Returning to equation (17) and using D_e as the average value of $D(x)$ over the most exposed portion of the member, and noting that $\phi(x)_{\max} = 1$ in all cases, we have

$$\delta_{\max} = \frac{\rho C_s D_e^3 [l_1/l_2]}{(4 \pi S)^2 \xi}$$

Taking $\xi = 0.01$ is reasonable (the Ontario Code takes $\xi = 0.0075$ for steel, $\xi = 0.015$ for concrete, and AASHTO takes .0005)
 $\rho = 0.002378$ slugs/ft³.

(d) Example: Circular steel pipe, 15.5 lb/ft; 6 inches in diameter; thickness 0.25 inches; member 100 feet long; end conditions somewhere between pin-ended and built-in:

$$\delta_{\max} = \frac{(0.002378)(0.7)(1/2)^3(1.3)(100)(32.2)}{[(4)(\pi)(0.2)]^2(0.01)(1550)} = 0.0089\text{Ft.}$$
$$= 0.11 \text{ in.}$$

The Ontario Code makes an in-depth study of the vortex excitation phenomenon that, it is felt, cannot be fully justified given the uncertainties present in the real field situation. More particularly, the use of the theory here presented, based upon equation (2) which itself is only a reasonable approximation.

In view of the preceding discussion and the apparent fact that oscillatory amplitudes due to vortex shedding are fairly small in any

event, the following simplified calculation of expected maximum amplitude is offered.

$$\delta_{\max} = \frac{\rho C_s D_e^3 [1.5 L/M_0]}{(4 \pi S)^2 \xi} = \frac{\rho C_s D_e^3 (1.5)}{(4 \pi S)^2 \xi w} \quad (18)$$

where:

δ_{\max} = peak deflection: at center if member is supported at both ends or at tip if member is cantilever

ρ = 0.002378 slugs/ft³

D_e = average member diameter, ft (In some cases the value of D_e should probably be the average of the "important" part of the member exposed to the wind.)

L = member length

M_0 = total member weight/g

S = Strouhal number

S = 0.20 for circular section

S = 0.11 for square section

S = 0.15 for other shapes

C_s = Lift coefficient

C_s = 0.7 for circular section

C_s = 0.85 for other sections

ξ = damping coefficient = 0.01

To calculate the internal stress (presumably for fatigue purposes) in the member, the member is loaded with a uniform spanwise load that gives the same maximum deflection δ_{\max} .

There does not appear to be a real need to ascertain the natural frequency of the structural member nor the wind velocity that would excite this frequency.

Compared to the deflections due to other wind and dead loads, those due to vortex excitation appear relatively low.

V. COMPUTER ANALYSES

All static and dynamic analyses performed in this report to compute deflections, forces, and frequencies were done using SAP-80, a microcomputer based three-dimensional analysis program based upon the stiffness method and written by Computers and Structures, Inc. The limitations of small deflections, linear material behavior, and elastic behavior are inherent in the procedure. This program has been in wide use for many years and the results were satisfactorily compared with computations by others using other programs, including those run on main frame computers.

Three configurations of the continuous, radial transition spanning sign structures and one of the tapered, articulated spanning sign structures were considered. For the radial type spans of 70 feet, 106 feet, and 142 feet were used while a span of 100 feet was used for the tapered structure. Appropriate member sizes were used to reflect the needs for stress adequacy. Both models used 31 nodes and 30 three dimensional frame elements. The column bases of all four frames were assumed fixed against any rotation or any translation, with the remaining degrees-of-freedom unconstrained.

1. DEAD LOAD ANALYSIS

Dead load analyses of both the tapered articulated and the radiused type of spanning sign support structures agreed closely with results obtained

by others. Since this procedure and the results obtainable utilized usual, routine procedures, it will not be discussed in detail in this report.

2. ALONG WIND STATIC ANALYSIS

As in the previous section, the analysis procedure used for the determination of stresses and deflections due to static wind pressure is well agreed upon. The determination of the appropriate coefficients for the effect of drag, exposure, and gusting have received wide attention and can be found in several sources^{2,3,5}. Sample calculations showed acceptable agreement with calculations by others⁶.

3. NATURAL FREQUENCY DETERMINATION

Although the development in Section IV shows that a reasonable determination of the behavior of sign structures can be obtained without knowledge of the natural frequency of the structure, some interesting characteristics of the systems were observed and will be reported here. Also, some discrepancies had been noted in frequencies determined by others and it was felt that the differences should be resolved to satisfy the validity of other calculations due to dead load and static wind pressure.

Each frame was analyzed using three different sign configurations: 1) no concentrated masses from signs; 2) all concentrated masses from signs placed at midspan; and 3) all concentrated masses from signs distributed across half the overhead span. It was felt that this should provide bounds on

the possible frequencies. The signs consisted of four 4' x 5' signs and one 2' x 5' sign, each weighing 10 pounds per square foot of sign surface area. The variation in the natural frequencies for the first ten modes are shown in figures 3 through 6. The primary mode of vibration for all of the cases was an out-of-plane mode with the vibration occurring normal to the plane of the sign structure. The remaining modes then alternate between in-plane modes and out-of-plane modes for the first ten mode shapes. For comparison the primary periods and frequencies of all models are tabulated in Table 1.

In both the figures and the table ADOT Frames refer to the radiused continuous structures and VALM refers to the articulated tapered structure.

ADOT Frame 1 - 70' Span

Frequency Variation

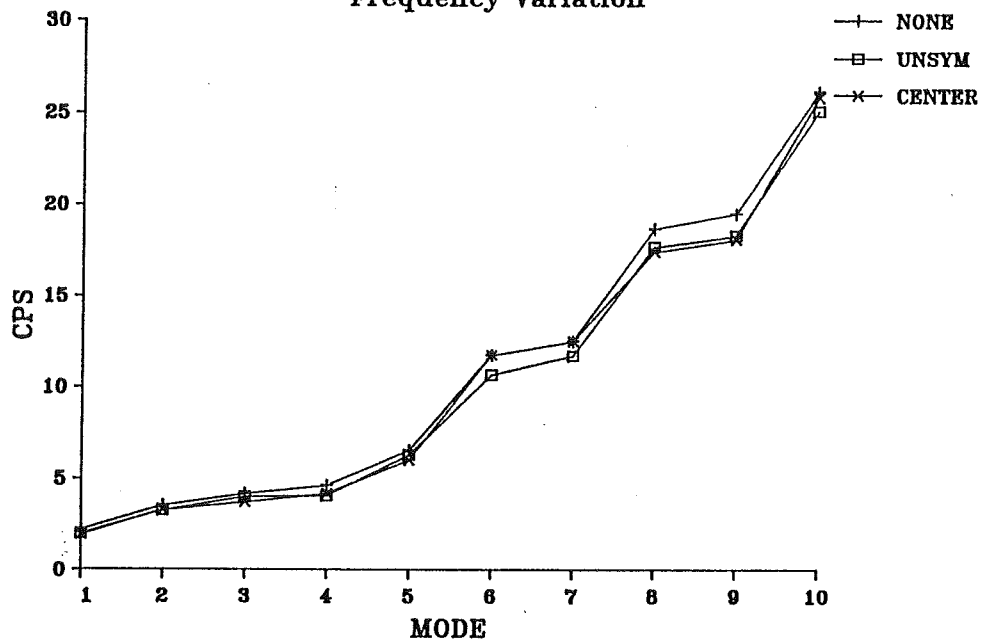


FIGURE 3

ADOT Frame 3 - 106' Span

Frequency Variation

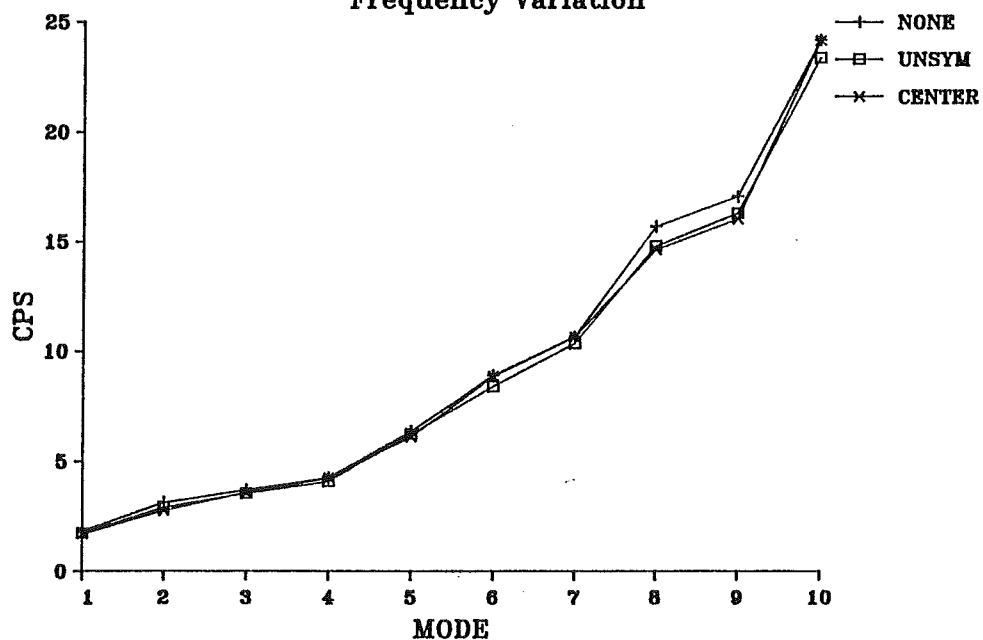


FIGURE 4

ADOT Frame 5 - 142' Span
Frequency Variation

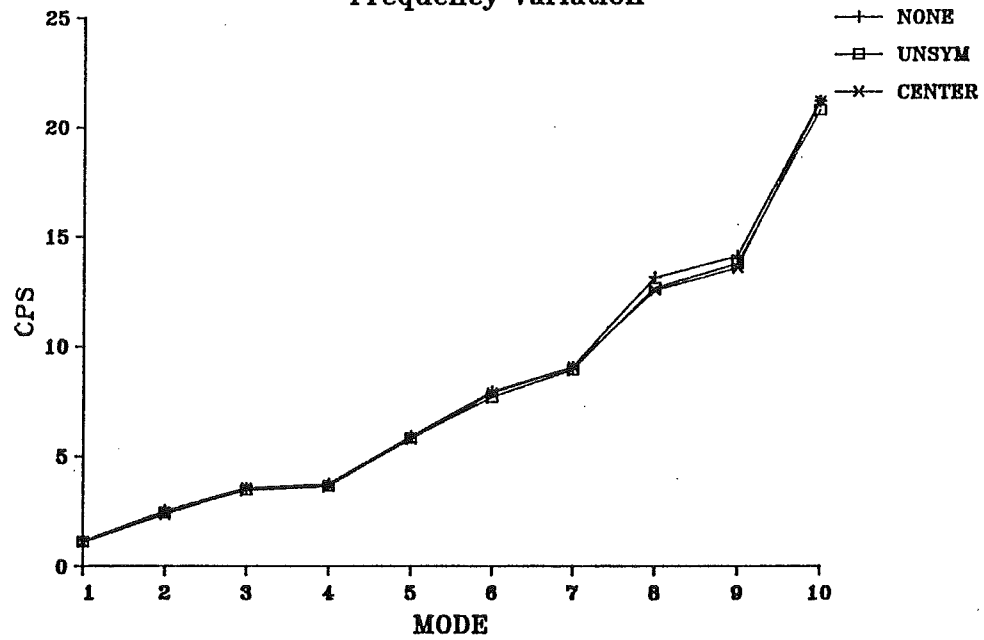


FIGURE 5

VALM Frame - 100' Span
Frequency Variation

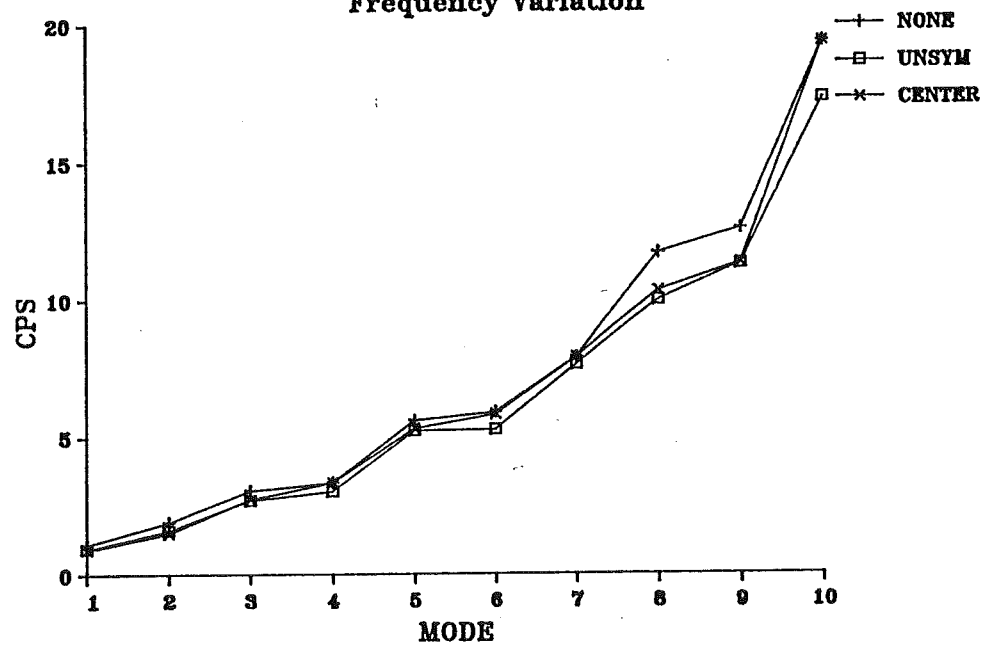


FIGURE 6

STRUCTURE	TIME PERIOD (sec)		
	NONE	UNSYM	CENTER
ADOT Frame 1	.4524	.5069	.5217
ADOT Frame 3	.5365	.5721	.5881
ADOT Frame 5	.8569	.8876	.9039
VALM Frame	.8946	1.0536	1.1021

STRUCTURE	NATURAL FREQUENCY (cps)		
	NONE	UNSYM	CENTER
ADOT Frame 1	2.2106	1.9729	1.9167
ADOT Frame 3	1.8640	1.7481	1.7004
ADOT Frame 5	1.1670	1.1267	1.1063
VALM Frame	1.1178	0.9491	0.9074

TABLE 1

It can be seen from the curves that there is very little variation in the frequencies of any of the frames when the masses and/or their location are changed. Also, from Table 1 it can be seen that, although there is some variation between the primary modes of the various configurations, the values are of the same order and that they are affected in very like manner by the variation of mass location.

Despite the difference in connection continuity and the difference in cross-section variability, these structures responded sufficiently alike to provide confidence that the simplified models of Section IV can reasonably represent a variety of real structures.

VI. SUMMARY

In compliance with the tasks outlined in the INTRODUCTION the following has been investigated and reported herein:

Task 1. Computerized searches were made of the data bases considered likely to yield relevant publications. In addition, published material not of a form to appear in the research data bases was sought by personal contact with involved persons. Several design codes were reviewed for their relevance.

Task 2. The significant portions of the most important material located under Task 1 has been reviewed in Section II of this report. In addition, the overall problem was reviewed with annotated comments on both the current AASHTO Specifications and selected reference material.

That the $d^2/400$ requirement is not appropriate as a cross-wind oscillation criterion was first criticized in 1971 by Pelkey⁴.

An unsuccessful attempt was made to identify instances of problems with cross-wind oscillation of span-type monotube structures. There appears to be a record of very good performance of these structures despite their not meeting the $d^2/400$ requirement.

There are at least two distinct configurations of span-type structures being designed and erected. These are the tapered, partially articulated and the continuous radial transition types. It appears that a generalized solution should be sought to best serve the industry.

Under section III. REVIEW OF PREVIOUS RELEVANT PUBLICATIONS several comments were made on selected sources.

The Ontario Highway Bridge Design Code and Commentary is a most complete and detailed source of design technology as well as references. It goes beyond what we have normally thought of as "Design Specifications". It includes a mathematically and technologically sophisticated procedure for analyzing all transportation related structures due to wind loads, including structures of the type being considered in the present investigation. If it was considered important, a fairly rigorous analysis of each sign structure could be executed using these methods.

Several elements of the Static and Dynamic Behavior of Monotube Span-type Sign Structures are worth noting. They felt that their findings were limited by their assumptions to solutions for wind velocities of twenty-nine miles per hour or less. This, in conjunction with the maximum of twenty-three miles per hour encountered on the field instrumented structures, limits the apparent usefulness of the data. They did, however, do extensive stress calculations and came to the conclusion that the stresses incurred during oscillation on the span-type structure are small compared to the stresses caused by usual gravity and static wind pressure loading. Also, the instrumentation phase of their work showed very little inclination of the structures to resonate under any of the wind velocities encountered, even if near a calculated natural frequency of the structure.

Task 3. The current state of designing and erecting monotube span-type structures is limited by lack of a procedure within AASHTO to facilitate reasonable, economical designs. The reason for this has been the lack of a logical procedure for the determination of the tendency to resonate and to determine the stresses occurring during cross-wind vibration. In Section IV of this study procedures are offered that solve this problem, including a detailed development and commentary.

Task 4. The recommendations required in Task 4 are presented in the next section, VII. CONCLUSIONS AND RECOMMENDATIONS.

VIII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The standard structural design practices for the determination of adequacy of the structure due to gravity and wind pressure loadings should continue to be employed. These procedures are well documented and do not vary significantly from source to source. The simplified process of using the maximum drag coefficient is recommended instead of computing Reynolds number (and thence a corresponding drag coefficient).

The standard procedure of limiting the dead load deflection should continue. It is imperative to realize, however, that it is not done to satisfy the cross-wind needs. The determination of a reasonable limitation on dead load deflection is arbitrary since the structure can be cambered to eliminate the visual "sag". It is common practice to camber the spanning member at least equal to the dead load deflection for aesthetic reasons. The University of Arizona report⁶ recommends a limiting dead load deflection of span/150. The traditional limit of $L/100$, to assure that the structure remains within the small deflection range and within the elastic limit, is probably an adequate stipulation.

A procedure has been developed in this report that can be used for the design of span-type as well as cantilever sign structures. This procedure, utilizing a few rational assumptions -the most important of which is that resonating lock-in has occurred, eliminates the need to know either the wind

velocity or the natural frequencies of the structure. The relative simplicity of the elements and of the cross-section facilitated the development of this proposed procedure. It should be noted, however, that some of the same assumptions are implied in the more cumbersome procedure outlined in the Ontario Code.

This study provides both an intermediate procedure that can be tailored more specifically to a structure as well as a simplified equation that takes advantage of the small variation that occurs when some of the parameters are varied. The simplified formula is identified as equation (18) in the text and a description of the parameters and their values follow at the end of section IV. The sample calculations that have been performed indicate that the maximum cross-wind deflection will be very small thus negating resonance as a concern except as it might result in fatigue problems. This procedure replaces: 1) the necessity of calculating the natural frequency of the structure, 2) the arbitrary selection of the vortex-shedding frequencies, and 3) the attempt to determine the lock-in tolerances.

The primary use of this procedure is to determine the deflections that occur during cross-wind oscillation so that maximum stresses can be estimated (Section IV demonstrates this calculation). This will provide a basis by which to evaluate the possibility that stresses might exceed the reduced permissible stress levels resulting from repeated cycling (fatigue).

Since a sign structure will ordinarily have an irregular silhouette it is highly unlikely to experience any vortex shedding unless it has a long stretch of uniform exposed pipe, in which case the preceding procedure is most applicable.

B. RECOMMENDATIONS

It is not recommended to perform wind-tunnel tests on either of the span-type structures discussed in this study. Analytic modeling is felt to be sufficiently accurate to predict the behavior within normal engineering design standards.

Although the equations developed in this study apply to cantilever sign support structures as well as to span-type, it is felt that additional work should be done on the cantilever structure because of its reported susceptibility to fatigue problems. There are several possible structural and/or aeroelastic modifications that may be able to alleviate the difficulties, however they were considered outside the scope of this investigation.

REFERENCES

1. American Association of State Highway and Transportation Officials (AASHTO) Bridge Design Specifications, Washington, D.C.
2. Standard Specifications for Structural Supports for Highway Signs, Luminaires, and Traffic Signals (1985), American Association of State Highway and Transportation Officials, Washington, D.C.
3. American National Standard for Minimum Design Loads for Buildings and Other Structures, ANSI A58.1-1982, American National Standards Institute, Inc.
4. Long Span Overhead Sign Structures, by Ron E. Pelkey, Deliuw Cather and Company of Canada, Presented at the 50th Annual Meeting of the Highway Research Board, January 1971.
5. 1983 Ontario Highway Bridge Design Code and Commentary, Toronto, Ontario.
6. Static and Dynamic Behavior of Monotube Span-type Sign Structures, Arizona Transportation and Traffic Institute, College of Engineering, University of Arizona, June 1985.
7. Field Testing of Monotube Span-type Sign Structures, Arizona Transportation and Traffic Institute, College of Engineering, University of Arizona, March, 1986.
8. Wind Effects on Structures, 2nd Edition by E. Simiu and R.H. Scanlan, pub by Wiley-Interscience, 1986.